

Magnetic Properties of Materials:

✓ Langevin's theory of diamagnetism:

we consider a atom of diamagnetic material in external magnetic field.

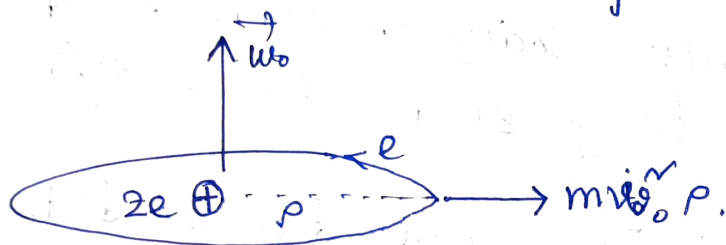
An uniform precession of the orbit about the dirⁿ of line of force takes place.

Now, the moving electron is equivalent to a current. The current associated magnetic moment,

$$\mu = IA$$

where, I = current

A = area of orbit.



Such current contributions from every electron orbit in the atom are in the same dirⁿ. so the effect from the various orbit added up.

In absence of the field, the circulation of current in random direction for each electron and cancel each other.

The amount of magnetic material moment opposite to that the field applied. and gives raise to a negative susceptibility.

To Find the resultant magnetic moment we consider the force on an electron of charge $-e$, mass m moving in a circular orbit of radius ρ .

In the absence of the external magnetic field,

we have
$$m\omega_0^2 \rho = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{\rho^2}$$

$$\therefore \omega_0 = \sqrt{\frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{\rho^3 m}}$$

where, ω_0 = frequency of rotation in the absence of field.

In the presence of the magnetic field B applied perpendicular to the plane of orbit, the frequency of rotation is changed. This is because an electron moving is equivalent to a current and there is flux. under the action of the magnetic moment field the change in flux and this change appears as a change in the frequency of rotation.

If the frequency in this case is ω , then

$$m\omega^2 \rho = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{\rho^2} - e\rho\omega B$$

$$\text{or, } \omega^2 + \frac{eB\omega}{m} - \frac{e^2}{4\pi\epsilon_0 m \rho^3} = 0$$

solving,

$$\omega = \frac{-eB/m}{2} \pm \frac{\sqrt{(eB/m)^2 + \frac{4e^2\hbar^2}{4\pi\epsilon_0 m \rho^3}}}{2}$$

$$= -\frac{eB}{2m} \pm \sqrt{\omega_0^2 + \left(\frac{eB}{2m}\right)^2}$$

$$= \pm \omega_0 - \omega_L \quad \left| \quad \omega_L = \frac{eB}{2m} \right.$$

Thus the effect of the magnetic field B is to change ω_0 . $\pm$ signs implies the angular momentum frequency decreases the magnetic moment in the direction of the field are slowed down and those whose whose moments were anti-parallel are speed up.

The current for the electron due to its frequency change $e\hbar/2\pi m$ can be written as.

$$I = -e \times \frac{\omega_L}{2\pi}$$

$$= -e \times \frac{eB}{2m \cdot 2\pi}$$

Electronic magnetic moment

$$\mu_e = -\frac{e^2 \hbar}{4\pi m} (\pi \rho^2)$$

$$= -\frac{e^2 \hbar \rho^2}{4\pi} \quad [\mu = I\lambda]$$

The negative sign indicates that the induced magnetic moment is always in opposite to that of the applied magnetic field.

If an atom contains a large number of electron z with their orbits oriented in all possible direction, the total magnetic moment,

$$M_e = - \frac{ze^v B}{4m} \overline{pr}$$

$$M_e = - \frac{ze^v B}{6m} \overline{r^2}$$

$$\overline{p^2} = \frac{2}{3} \overline{r^2}$$

$\therefore$ If the number of atoms per unit volume is N then

$$M = - \frac{Nze^v M_0 H \overline{r^2}}{6m}$$

$\therefore$ Diamagnetic susceptibility

$$\chi_{dia} = + \frac{M}{H}$$

$$= - \frac{Nze^v \overline{r^2} M_0}{6m}$$

Langevin's theory of paramagnetic gas (classical)

classical theory of paramagnetic gas developed by Langevin's. The theory assumes the paramagnetic solids in terms of paramagnetic gas in which each particle is assumed to have a paramagnetic moment μ .

We consider the unit volume of a paramagnetic gas contains n particles. Applied magnetic field $= B$.

$$\therefore \text{potential energy of a particle} = -\mu B \cos \theta$$

where, $\theta =$ angle between magnetic axis of the particle and field direction.

$\therefore$ According to Maxwell Boltzmann distribution law, number of particles with their magnetic axis making angle θ with the field B is proportional to $e^{\mu B \cos \theta / k_B T}$

Number of particles whose magnetic axis fall within the solid angle $d\Omega$ between two cones of semi angle θ and $\theta + d\theta$

$$dN = K \cdot e^{\mu B \cos \theta / k_B T} \cdot d\Omega$$

$$\left| \begin{aligned} d\Omega &= \frac{2\pi (\mu \sin \theta) \mu d\theta}{4\pi} \\ &= 2\pi \sin \theta d\theta \end{aligned} \right.$$

$$d\Omega = \text{solid angle} = 2\pi \sin \theta d\theta$$

$\therefore$ Total number of particles in the gas

$$n = 2\pi K \int_0^\pi e^{\mu B \cos \theta} \sin \theta d\theta$$

where $\mu_B/k_B T$

$$n = -2\pi K \int_1^{-1} e^{ax} dx$$

putting $\cos\theta = x$

$$\therefore dx = -\sin\theta d\theta$$

$$= \frac{2\pi K}{a} (e^a - e^{-a})$$

$$= \frac{4\pi K}{a} \sinh a$$

∴ Magnetic moment per unit volume

$$M = \int_0^\pi \mu \cos\theta n d\theta$$

$$= \int_0^\pi \mu \cos\theta K e^{(\mu H \cos\theta)/k_B T} 2\pi \sin\theta d\theta$$

$$= -2\pi \mu K \int_1^{-1} x e^{ax} dx \quad \left| \begin{array}{l} x = \cos\theta \\ dx = -\sin\theta d\theta \end{array} \right.$$

$$= +2\pi \mu K \left[x e^{ax}/a + \frac{e^{ax}}{a^2} \right]_1^{-1}$$

$$= +2\pi \mu K \left[\frac{e^{-a}}{a^2} + \frac{e^{-a}}{a} - \frac{e^a}{a^2} + \frac{e^a}{a} \right]$$

$$= 2\pi \mu K \left[\frac{2\cosh a}{a} - \frac{2\sinh a}{a^2} \right]$$

$$\frac{M}{n} = \frac{\mu}{2\sinh a} \left[2\cosh a - \frac{2\sinh a}{a} \right]$$

$$= \ln \left[\cosh a - \frac{1}{a} \right]$$

$$\therefore M = \mu n L(a)$$

$$L(a) = \omega \left(ha - \frac{1}{a} \right)$$

= Langevin function

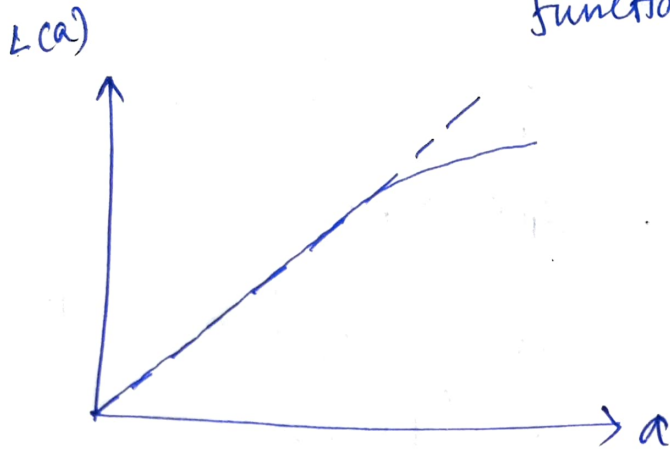


fig: plot of $L(a)$ against 'a'.

For case 1: For small value of a , the curve is linear and coincides with the tangent to the curve at the origin.

$$\therefore L(a) = a/3.$$

$$\therefore M = \frac{\mu n a}{3},$$

$$\therefore \frac{M}{\theta} = \frac{\mu n}{3} \cdot \frac{\mu_B}{k_B T}$$

$$\therefore \frac{M}{H} = \frac{\mu_B^2 n}{3 k_B T} = \chi.$$

Quantum theory of paramagnetism:

In quantum theory, since their magnetic moment are quantized, they can orient only at certain direction with respect to the magnetic field.

The possible component of magnetic moment ~~is~~ of each atom parallel to the field with total angular momentum equal to $M_J g \mu_B$

$$\text{where, } M_J = J, J-1, \dots, -J$$

potential energy of the atom $= -M_J g \mu_B B$ $g = \text{Lange-g-factor}$

According to Maxwell-Boltzmann distribution law, the number of atoms with a particular value of M_J will be proportional to $e^{M_J g \mu_B B / k_B T}$.

Now we consider unit volume containing n atoms placed in the magnetic field B .

$\therefore$ Magnetization along the magnetic field

$M = n \times \text{Statistical average magnetic moment per atom along } B$

$$M = n \times \frac{\sum_{M_J=-J}^{+J} M_J g \mu_B e^{M_J g \mu_B B / k_B T}}{\sum_{M_J=-J}^J e^{M_J g \mu_B B / k_B T}} \quad \dots (1)$$

Case: I At normal field strengths and ordinary temperature with $g=2$ and $J=1/2$, The value of the $M_J g \mu_B B / k_B T$ is very small

$$M = n \frac{g \mu_B \sum_{-J}^J M_J \left(1 + \frac{M_J g \mu_B B}{k_B T}\right)}{\sum_{-J}^J \left(1 + \frac{M_J g \mu_B B}{k_B T}\right)}$$

$$= n \frac{g \mu_B \cdot \frac{g \mu_B B}{k_B T} \cdot \frac{J(J+1)(2J+1)}{3}}{(2J+1)}$$

$$= \frac{n g^2 \mu_B^2 B J(J+1)}{3 k_B T}$$

$$\therefore \chi = \frac{M}{H}$$

$$= \frac{\mu_0 n \mu_B^2}{3 k_B T} g^2 J(J+1)$$

Case: II At intense fields and low temperature $M_J g \mu_B B / k_B T$ is not smaller than unity

$$M = n g \mu_B \frac{\sum_{-J}^J M_J e^{M_J \chi}}{\sum_{-J}^J e^{M_J \chi}}$$

$$\chi = \frac{g \mu_B B}{k_B T}$$

$$= ng_{\text{lib}} \frac{d}{dn} \left[\ln \sum_{-J}^J e^{M_J x} \right]$$

$$= ng_{\text{lib}} \frac{d}{dn} \left[\ln \cdot \frac{e^{Jx} \{1 - e^{-x(2J+1)}\}}{1 - e^{-x}} \right]$$

$$= ng_{\text{lib}} \frac{d}{dn} \left[\ln \frac{e^{(J+1/2)x} - e^{-(J+1/2)x}}{\sinh x/2} \right]$$

$$= ng_{\text{lib}} \cdot \frac{d}{dn} \left[\ln \frac{\sinh (J+1/2)x}{\sinh x/2} \right]$$

$$= ng_{\text{lib}} \left[\frac{2J+1}{2} \coth \frac{(2J+1)}{2} x - \frac{1}{2} \coth x/2 \right]$$

$$= ng_{\text{lib}} \ln(a).$$

- Discuss Weiss theory of spontaneous magnetization
- Deduce Curie Weiss law of ferromagnetism.

Weiss theory:

(a) A ferromagnetic substance contains a number of small regions called domains ($10^{15} - 10^7$ atoms) which are spontaneously magnetized. The magnitude of spontaneous magnetization of the specimen is determined by the vector sum of the magnetic moment of individual domains.

(b) Within each domain, the spontaneous magnetization is due to the existence of a molecular field which tends to produce a parallel alignment of the atomic dipoles. Further this internal magnetic dipole is equivalent to a magnetic field H_m , which is proportional to the magnetization M within a domain.

$$H_m = \lambda M$$

where λ = independent of temperature called Weiss constant.

Thus if H is the external magnetic field, the effective field acting on an ion or atom is given by

$$H_{eff} = H + \lambda M \quad \dots (1)$$

But as long as the spontaneous magnetization is concerned $H = 0$

$$\therefore H_{eff} = \lambda M$$

Now, if we consider the unit volume of a ferromagnetic substance contains n atoms placed in the magnetic field H ,

$$M = ngJ\mu_B B(a) \quad (2)$$

$$B(a) = \frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J}a\right) - \frac{1}{2J} \coth\left(\frac{a}{2J}\right)$$

$$a = \frac{g\mu_B B J}{k_B T}$$

$$= \frac{g\mu_B \lambda M J}{k_B T} \quad (3)$$

Now, the magnetic moment per unit volume corresponding to complete alignment of the magnetic moment parallel to the field.

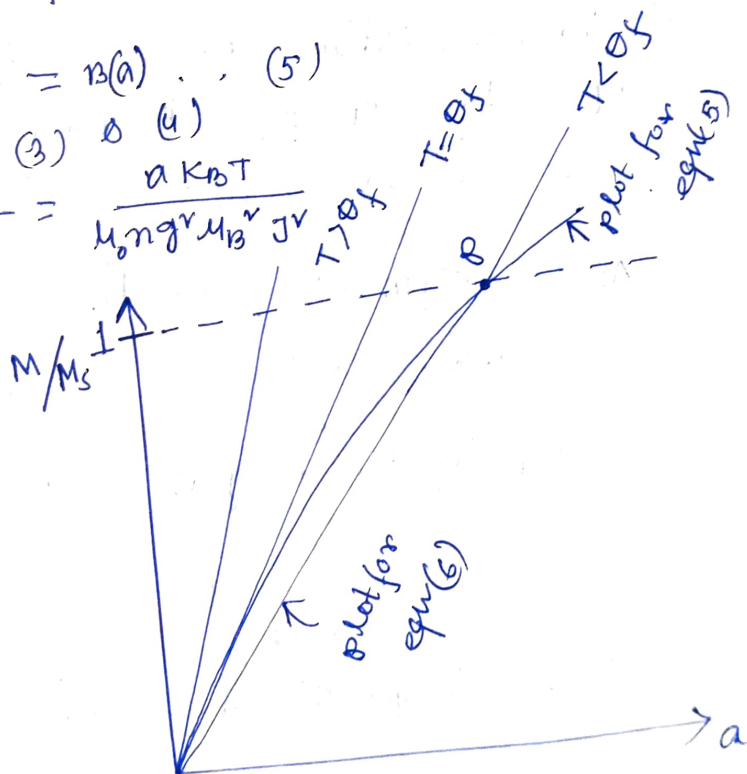
$$M_S = ngJ\mu_B \quad (4)$$

From eqn (2) & (4)

$$\frac{M}{M_S} = B(a) \quad (5)$$

From (3) & (4)

$$\frac{M}{M_S} = \frac{a k_B T}{\mu_B n g J \mu_B J}$$



Eq (b) represents a straight line passing through the origin. The slope of the line is proportional to T .

Since M satisfy eqn (5) & (6) both, value of M at a given temperature may be obtained from the point of intersection of the two corresponding M vs a curves.

Examining the curves, we get a point a for $T < \theta_f$ for which there is a nonvanish value of M although the external field is zero. The theory thus allows for spontaneous magnetization. It also follows that as the temperature rises.

There is no solution at $T = \theta_f$. Thus θ_f is the critical temperature known as the ferro magnetism.

To find the relation between θ_f and λ , we compare the slope of eqn (5) and (6) at $T = \theta_f$ and slope of eqn (5) at the origin.

At the origin $a \rightarrow 1$ ($a < 1$).

$$B(a) = \frac{J+1}{3J} a$$

Slope of the tangent at the origin curve

$$= \frac{J+1}{3J}$$

Slope of eqn (5) at $T = \theta_f$.

$$= \frac{k_B \theta_f}{\mu_0 n g^2 \mu_B^2 \lambda J^2}$$

$$\therefore \frac{J+1}{3J} = \frac{k_B \theta_f}{ng^v \mu_B^v \mu_0 J^v}$$

$$\therefore \frac{3k_B \theta_f}{J} = ng^v \mu_B^v J(J+1)$$

$$\text{or, } \frac{\theta_f}{J} = \frac{\mu_0 ng^v \mu_B^v J(J+1)}{3k_B}$$

$$= n \mu^v \quad \left| \begin{array}{l} \mu = \text{Total magnetic} \\ \text{moment per} \\ \text{atom.} \end{array} \right.$$

$$\therefore \boxed{\theta_f \propto J}$$

$$M/M_s = \frac{J+1}{3J} \cdot \left(\frac{T}{\theta_f} \right) a.$$

At $T > \theta_f$, there is no spontaneous magnetization when an external magnetic field B is applied.

For fields low enough

$$B(a) = \frac{J+1}{3J} a.$$

$$M = ng \mu_B (J+1) \cdot \frac{a}{3}.$$

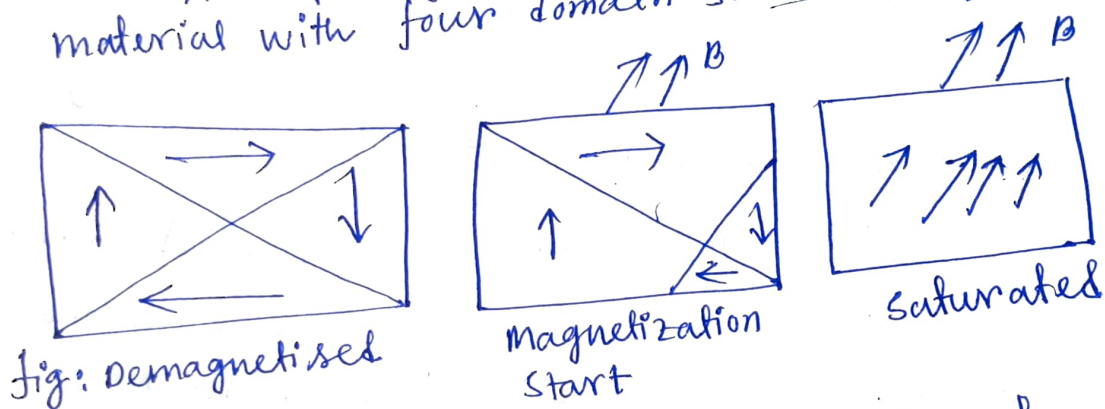
$$a = \frac{g \mu_B \mu_0 (H + \lambda M) J}{k_B T}$$

Hysteresis using Domain Theory:

A ferromagnetic crystal contains large number regions in which all dipole spin is arranged in same direction, called domain. Each domain contained atoms (10^9 - 10^{15}) atoms.

To explain the hysteresis Weiss developed domain Hypothesis, according to Weiss hypothesis with in each domain all the dipoles spin are arranged in same direction, so the domains are self magnetised to saturation but macroscopically magnetization vary from zero to saturation.

A completely demagnetised ferromagnetic material with four domain shown in figure



If a gradually increasing external field is applied to magnetic moment specimen in a arbitrary direction, the domain with a magnetic moment along the same direction of the external will grow quickly compare to unfavourable domain due to the motion of domain wall. As the field is increased further the magnetization increase to saturation due to certain rotation of unfavourable domain, so the whole specimen is oriented in same direction of applied field.

The specimen reaches maximum magnetization (M_s).

When the external magnetic field increase further the magnetization does not increase.

If we decrease the magnetic field, the magnetization decrease in different path and for a zero field, the specimen contains a amount of magnetization called residual magnetization. A finite reverse field $-H_c$ required to demagnetised the specimen, known as coercive field.

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write down the validity of Weidemann-Frentz law.

= According to Weidemann-Frentz law,

$$\frac{\sigma_T}{\sigma} = LT, \text{ where, } \sigma_T = \text{Thermal conductivity}$$
$$\sigma = \text{electrical conductivity}$$
$$L = \text{Lorentz No.}$$
$$T = \text{Absolute Temperature.}$$

This relation only valid at high Temperature. At low temperature $T \ll \theta_D$, the L value decreases due to the difference of freetime of collision in two type of conductivity.

"Electrical conductivity of a metal decreases with temperature but in semiconductor it increases with temperature"—Explain.

= From Weidemann-Frentz equation,

$$\frac{\sigma_T}{\sigma} \propto T$$

Where,

σ_T = Thermal conductivity

σ = Electrical conductivity

So, the electrical conductivity decreases as the increase in Temperature.

In semiconductor (intrinsic),

$$\sigma(T) = \sigma(0) e^{-\frac{E_g}{2kT}}$$

where, $\sigma(T)$ = conductivity at Temperature T

$\sigma(0)$ = conductivity at OK.

E_g = Band gap.

So, in semiconductor with the increase in temperature conductivity increases.

In what condition Langerin's Quantum theory of para-magnetism reduces to Langerin's classical theory.

= At normal Temperature and normal field strength,

$$\chi_{\text{para}} = \frac{n \mu_B^2 \mu_0}{3k_B T} g^2 J(J+1)$$

$$g = 1, \quad J = \frac{1}{2}$$

$$\chi_{\text{para}} = \frac{n \mu_B^2}{k_B T}$$

which is same as classical Langevin theory of paramagnetism.

"The Bandgap in semi-conductors decreases with the increase in temperature" - Explain.

= The energy band-gap at $T^\circ\text{K}$ of an intrinsic semiconductor given by

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{\beta + T}$$

where
 $E_g(0)$ = Band gap at 0°K

α, β are constant.

From above equation it is clear that Band gap in semiconductor decreases with the increase in temperature.

